Quiz 2007

This is a quiz designed for fifth year post-primary students. Students work in teams of 3 and have an hour to complete as many problems as possible.

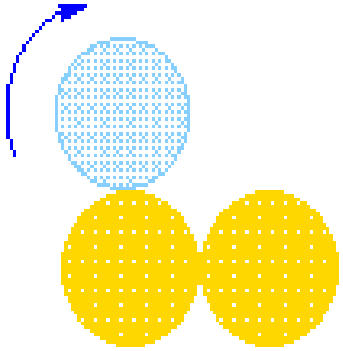
Many of the questions here are taken from www.nrich.maths.org

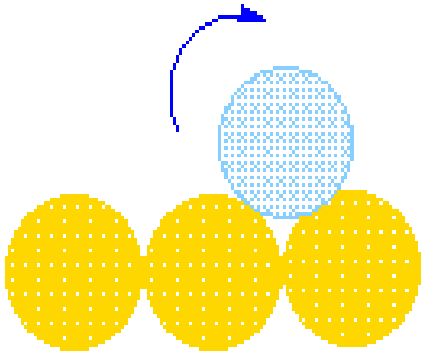
1. An engineer is directed to a faulty signal, one quarter of the way through a tunnel. Whilst there, he is warned of a train heading towards the tunnel entrance. The engineer can run at 12 mph and can either run back to the tunnel entrance or forward to the exit. In either case, the engineer and train would meet the entrance or exit at the same time. At what speed is the train travelling?

2. Why 24?

Take any prime number greater than 3, square it, subtract one and divide by 24. Prove that there is no remainder.

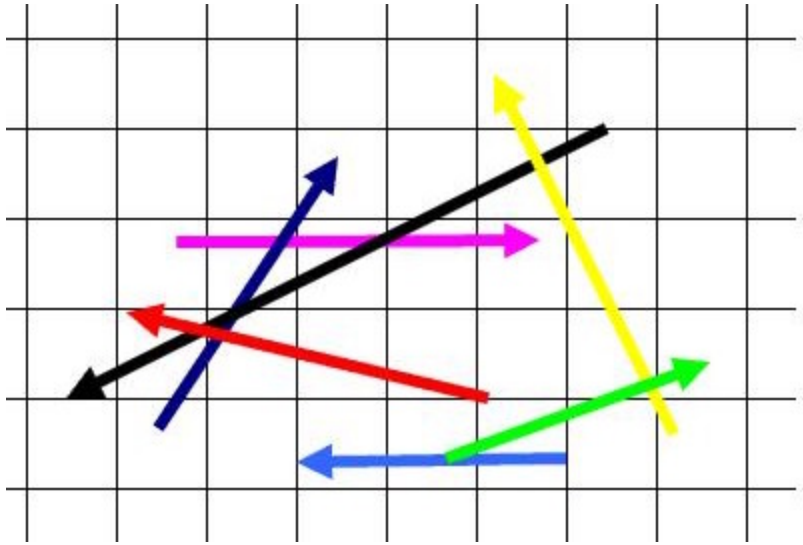
3. A blue coin rolls round two yellow coins which touch. The coins are the same size.
How many revolutions does the blue coin make when it rolls all the way round the yellow coins?
Investigate for a line of 'n' coins (starting at n = 1)



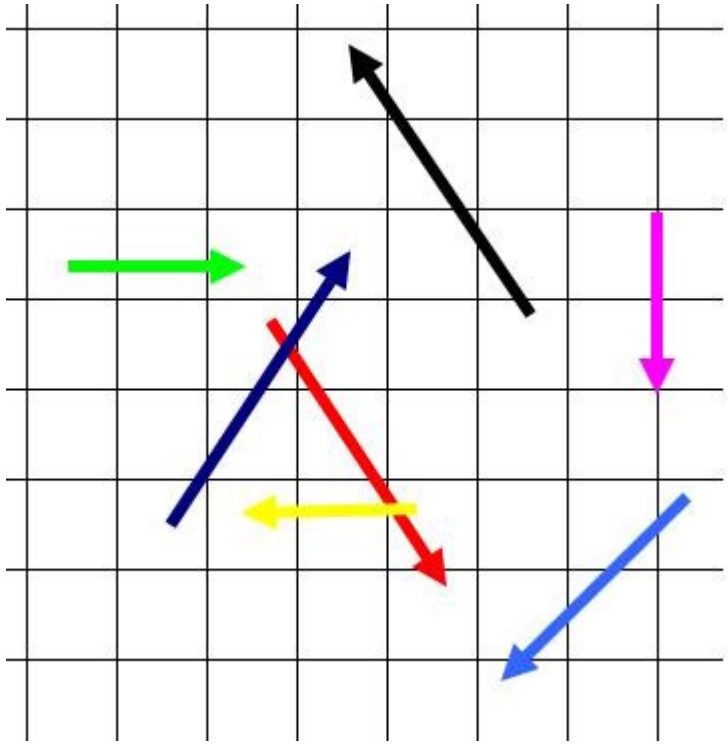


4. In each grid can you find a closed loop of vectors?
In each case is the closed loop unique?
You can prove your assertions using a convincing visual argument.

Grid 1:



Grid 2:



5. Honeybees and Family trees

Honeybees live in a colony called a hive and they have an unusual Family Tree. In a colony of honeybees there is one special female called the **queen**. There are many **worker** bees who are female too but unlike the queen bee, they produce no eggs. There are some **drone** bees who are male and do no work.
Males are produced by the queen's unfertilized eggs, so male bees only have a mother but no father! All the females are produced when the queen has mated with a male and so have two parents. So female bees have 2 parents, a male and a female whereas male bees have just one parent, a female.

A male drone bee has **1** parent, a female; **2** grand-parents, since his mother had two parents, a male and a female; **3** great-grand-parents as his grand-mother had two parents but his grand-father had only one.

How many great-great-grand parents did he have?

How many great-great-great-grand parents did he have?

Draw a family tree for a male.

Draw a family tree for a female bee.

What do you notice?

6. The Fibonacci numbers can be described by $F_1=1$, $F_2=1$, $F_{n+2}= F_{n+1} + F_n$

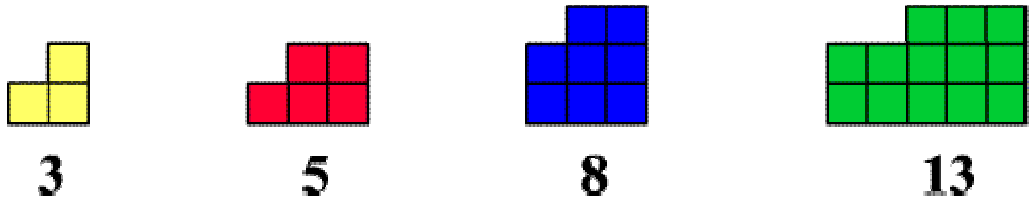
Write down the first 10 terms of this sequence.

What can you say about (F_{n-1} x F_{n+1})? Can you explain?

For what values of n is F_n even? Why?

For what values of n is F_n divisible by 3? Why?

7. A gnomon is a rectangle with a rectangle cut out of one corner. (Can you think of a number that does not have a gnomon?) Each Fibonacci number has its own gnomon.



Draw gnomons for the next four Fibonacci numbers.

Study the shapes of the gnomons for the alternate Fibonacci numbers.

Can you make a conjecture about alternate Fibonacci numbers?

Can you prove it?

8. Continued fractions are written as fractions within fractions which are added up in a special way, and which may go on for ever. For example, $a = \dfrac{1}{2+\dfrac{3}{4}}$ what about the sequence of fractions

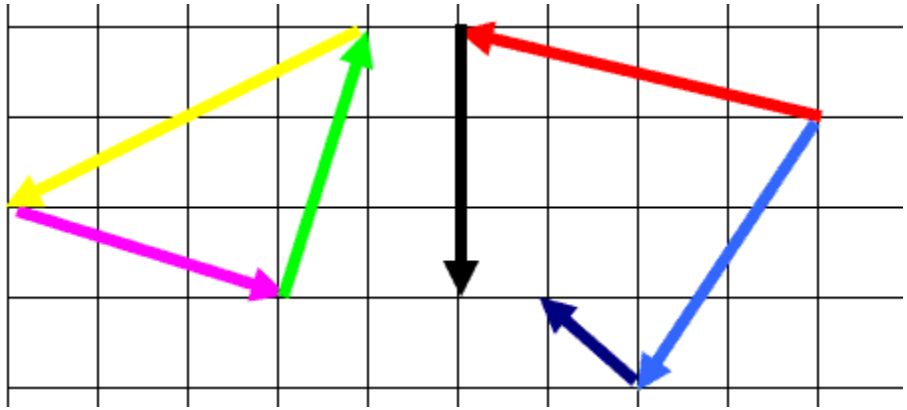
$$1, \quad \frac{1}{1+1}, \quad \frac{1}{1+\dfrac{1}{1+1}}, \quad \frac{1}{1+\dfrac{1}{1+\dfrac{1}{1+1}}}, \quad \frac{1}{1+\dfrac{1}{1+\dfrac{1}{1+\dfrac{1}{1+1}}}}, \ldots$$

What happens if the continued fraction sequence goes on for ever, what will it eventually equal?

Information for Question 4.

In 2D, vectors may be thought of as arrows with a fixed length and direction. The place at which the arrow starts is not important, so they may be translated around the plane without affecting the value of the vector.

This allows us to add and subtract vectors visually: if the vectors can be translated so as to form a closed circuit then the vector sum is zero. In this diagram the vectors on the left have zero sum, whereas the vectors on the right don't have zero sum.



Fibonacci Sequences in Nature

http://www.mcs.surrey.ac.uk/Personal/R.Knott/Fibonacci/fibInArt.html#parthen for Fibonacci in architecture